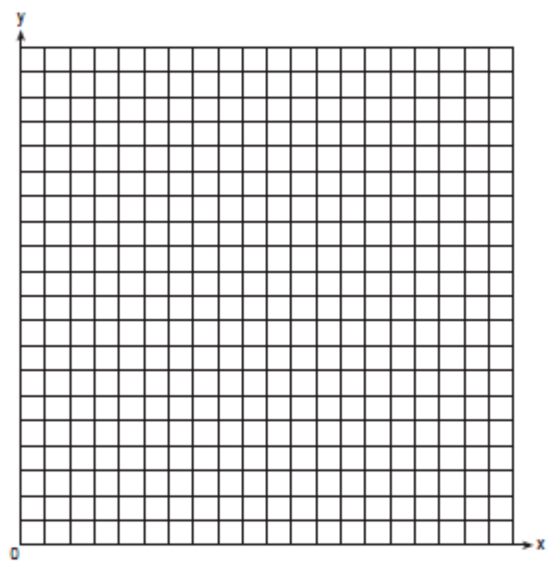
Name: ________________________________

G.G.69: Quadrilaterals in the Coordinate Plane: Investigate, justify, and apply the properties of quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas

1 The coordinates of quadrilateral *PRAT* are $P(a,b)$, $R(a,b+3)$, $A(a+3,b+4)$, and $T(a+6,b+2)$. Prove that $\overline{RA}$ is parallel to $\overline{PT}$.

2 Ashanti is surveying for a new parking lot shaped like a parallelogram. She knows that three of the vertices of parallelogram *ABCD* are $A(0,0)$, $B(5,2)$, and $C(6,5)$. Find the coordinates of point *D* and sketch parallelogram *ABCD* on the accompanying set of axes. Justify mathematically that the figure you have drawn is a parallelogram.



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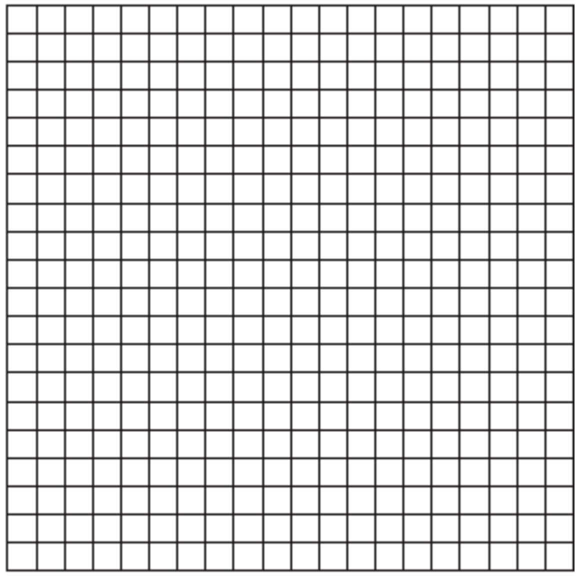
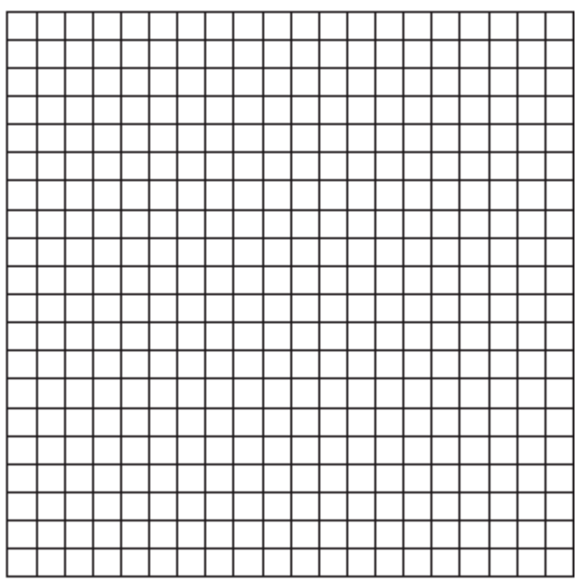
3 Given: $A(-2,2)$, $B(6,5)$, $C(4,0)$, $D(-4,-3)$
Prove: *ABCD* is a parallelogram but not a rectangle. [The use of the grid is optional.]



4 The coordinates of quadrilateral *ABCD* are $A(-1,-5)$, $B(8,2)$, $C(11,13)$, and $D(2,6)$. Using coordinate geometry, prove that quadrilateral *ABCD* is a rhombus. [The use of the grid is optional.]



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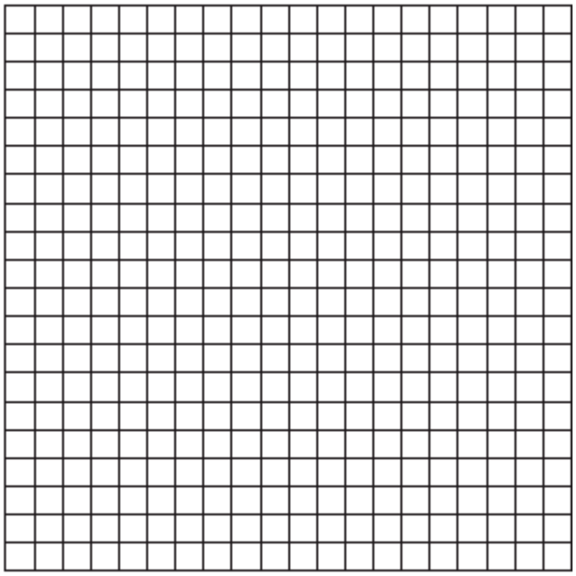
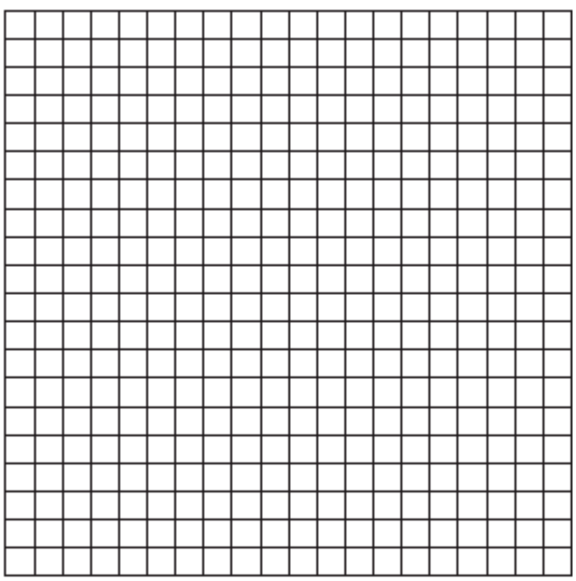
5 Quadrilateral *ABCD* has vertices $A(2,3)$, $B(7,10)$, $C(9,4)$, and $D(4,-3)$. Prove that *ABCD* is a parallelogram but *not* a rhombus. [The use of the grid is optional.]



6 Jim is experimenting with a new drawing program on his computer. He created quadrilateral *TEAM* with coordinates $T(-2,3)$, $E(-5,-4)$, $A(2,-1)$, and $M(5,6)$. Jim believes that he has created a rhombus but not a square. Prove that Jim is correct. [The use of the grid is optional.]



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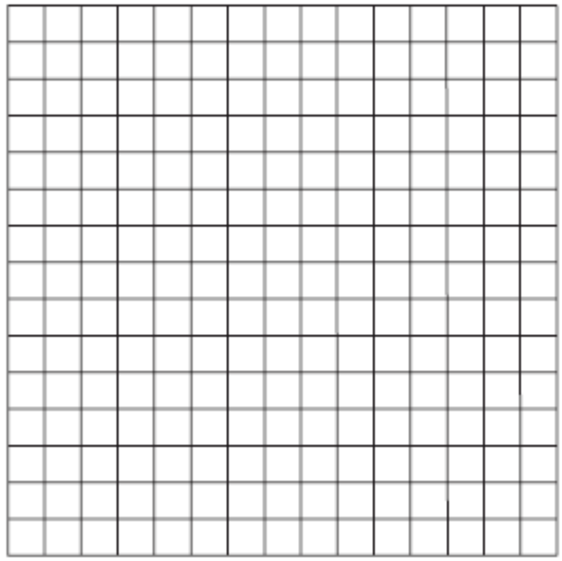
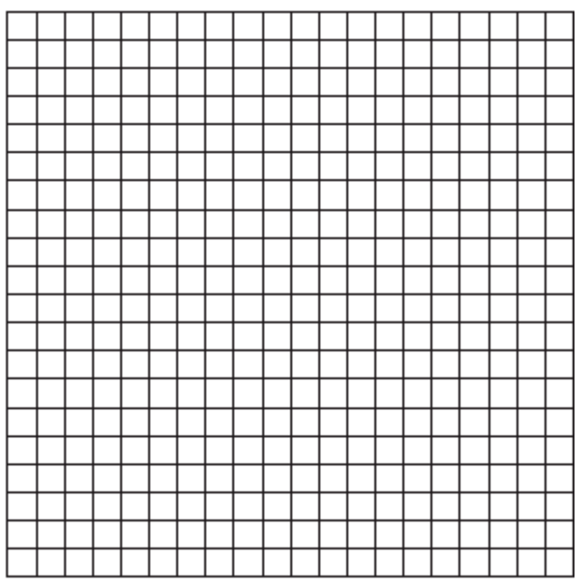
7 Given: $A(1,6)$, $B(7,9)$, $C(13,6)$, and $D(3,1)$
Prove: *ABCD* is a trapezoid. [*The use of the accompanying grid is optional*.]



8 Quadrilateral *KATE* has vertices $K(1,5)$, $A(4,7)$, $T(7,3)$, and $E(1,-1)$.
a Prove that *KATE* is a trapezoid. [The use of the grid is optional.]
b Prove that *KATE* is *not* an isosceles trapezoid.



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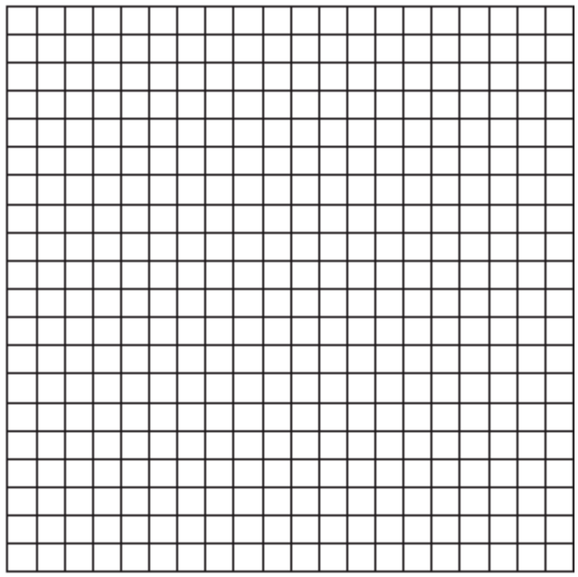
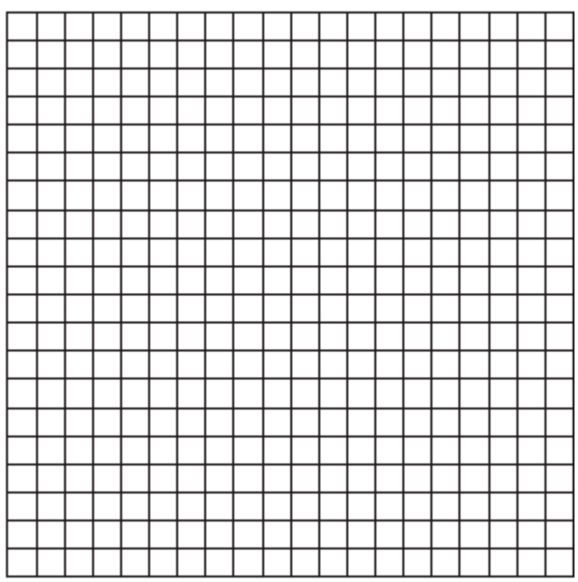
9 The coordinates of quadrilateral *JKLM* are $J(1,-2)$, $K(13,4)$, $L(6,8)$, and $M(-2,4)$. Prove that quadrilateral *JKLM* is a trapezoid but *not* an isosceles trapezoid. [The use of the grid is optional.]



10 Given: $T(-1,1)$, $R(3,4)$, $A(7,2)$, and $P(-1,-4)$
Prove: *TRAP* is a trapezoid.
TRAP is not an isosceles trapezoid.
[The use of the grid is optional.]



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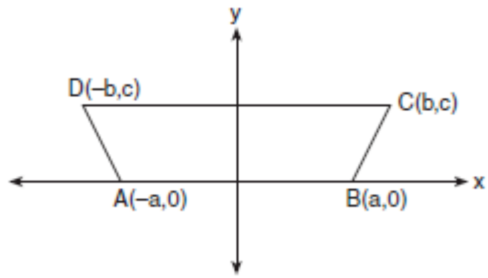
11 In the accompanying diagram of *ABCD*, where $a \neq b$, prove *ABCD* is an isosceles trapezoid.

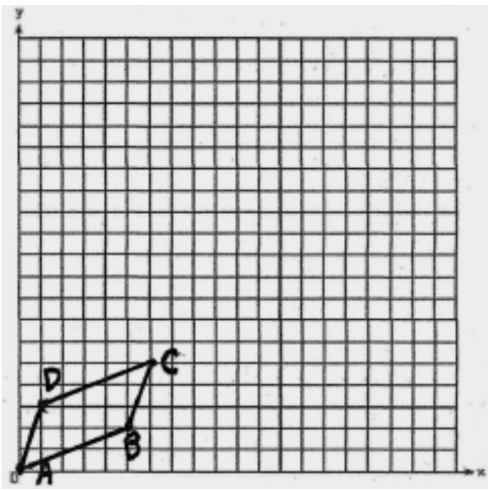


1 ANS:
$m_{\overline{RA}} = \frac{(b+3)-(b+4)}{a-(a+3)} = \frac{-1}{-3} = \frac{1}{3}$. Because $\overline{RA}$ and $\overline{PT}$ have equal slopes, they are parallel.

$m_{\overline{PT}} = \frac{b-(b+2)}{a-(a+6)} = \frac{-2}{-6} = \frac{1}{3}$

REF: 060824b

2 ANS:



Both pairs of opposite sides of a parallelogram are parallel. Parallel lines have the same slope. The slope of side $\overline{BC}$ is 3. For side $\overline{AD}$ to have a slope of 3, the coordinates of point D must be $(1,3)$. $m_{\overline{AB}} = \frac{2-0}{5-0} = \frac{2}{5}$ $m_{\overline{AD}} = \frac{3-0}{1-0} = 3$

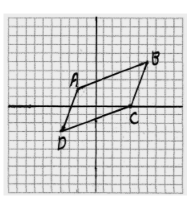
$m_{\overline{CD}} = \frac{5-3}{6-1} = \frac{2}{5}$ $m_{\overline{BC}} = \frac{5-2}{6-5} = 3$

REF: 080032a

3 ANS:
To prove that *ABCD* is a parallelogram, show that both pairs of opposite sides of the parallelogram are parallel by showing the opposite sides have the same slope: $m_{\overline{AB}} = \frac{5-2}{6-(-2)} = \frac{3}{8}$ $m_{\overline{AD}} = \frac{-3-2}{-4-(-2)} = \frac{5}{2}$

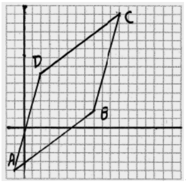
$m_{\overline{CD}} = \frac{-3-0}{-4-4} = \frac{3}{8}$ $m_{\overline{BC}} = \frac{5-0}{6-4} = \frac{5}{2}$

A rectangle has four right angles. If *ABCD* is a rectangle, then $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{CD}$, $\overline{CD} \perp \overline{AD}$, and $\overline{AD} \perp \overline{AB}$. Lines that are perpendicular have slopes that are the opposite and reciprocal of each other. Because $\frac{3}{8}$ and $\frac{5}{2}$ are not opposite reciprocals, the consecutive sides of *ABCD* are not perpendicular, and *ABCD* is not a rectangle.



REF: 060633b

4 ANS:



. To prove that *ABCD* is a rhombus, show that all sides are congruent using the distance formula: $d_{\overline{AB}} = \sqrt{(8-(-1))^2 + (2-(-5))^2} = \sqrt{130}$.

$$d_{\overline{BC}} = \sqrt{(11-8)^2 + (13-2)^2} = \sqrt{130}$$

$$d_{\overline{CD}} = \sqrt{(11-2)^2 + (13-6)^2} = \sqrt{130}$$

$$d_{\overline{AD}} = \sqrt{(2-(-1))^2 + (6-(-5))^2} = \sqrt{130}$$

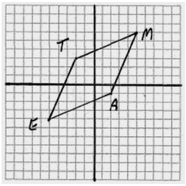
REF: 060327b

5 ANS:

$m_{\overline{AB}} = \frac{10-3}{7-2} = \frac{7}{5}$, $m_{\overline{CD}} = \frac{4-(-3)}{9-4} = \frac{7}{5}$, $m_{\overline{AD}} = \frac{3-(-3)}{2-4} = \frac{6}{-2} = -3$, $m_{\overline{BC}} = \frac{10-4}{7-9} = \frac{6}{-2} = -3$ (Definition of slope). $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{BC}$ (Parallel lines have equal slope). Quadrilateral *ABCD* is a parallelogram (Definition of parallelogram). $d_{\overline{AD}} = \sqrt{(2-4)^2 + (3-(-3))^2} = \sqrt{40}$, $d_{\overline{AB}} = \sqrt{(7-2)^2 + (10-3)^2} = \sqrt{74}$ (Definition of distance). $\overline{AD}$ is not congruent to $\overline{AB}$ (Congruent lines have equal distance). *ABCD* is not a rhombus (A rhombus has four equal sides).

REF: 061031b

6 ANS:



. To prove that *TEAM* is a rhombus, show that all sides are congruent using the distance formula: $d_{\overline{ET}}=\sqrt{(-2-(-5))^2+(3-(-4))^2}=\sqrt{58}$. A square has four right angles. If *TEAM* is a square,

$$d_{\overline{AM}}=\sqrt{(2-5)^2+((-1)-6)^2}=\sqrt{58}$$

$$d_{\overline{AE}}=\sqrt{(-5-2)^2+(-4-(-1))^2}=\sqrt{58}$$

$$d_{\overline{MT}}=\sqrt{(-2-5)^2+(3-6)^2}=\sqrt{58}$$

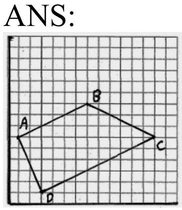
then $\overline{ET}\perp\overline{AE}$, $\overline{AE}\perp\overline{AM}$, $\overline{AM}\perp\overline{AT}$ and $\overline{MT}\perp\overline{ET}$. Lines that are perpendicular have slopes that are opposite reciprocals of each other. The slopes of sides of *TEAM* are: $m_{\overline{ET}}=\frac{-4-3}{-5-(-2)}=\frac{7}{3}$ $m_{\overline{AE}}=\frac{-4-(-1)}{-5-2}=\frac{3}{7}$

$$m_{\overline{AM}}=\frac{6-(-1)}{5-2}=\frac{7}{3}\quad m_{\overline{MT}}=\frac{3-6}{-2-5}=\frac{3}{7}$$

Because $\frac{7}{3}$ and $\frac{3}{7}$ are not opposite reciprocals, consecutive sides of *TEAM* are not perpendicular, and *TEAM* is not a square.

REF: 010533b

7 ANS:

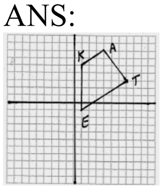


. To prove that *ABCD* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{AB}}=\frac{9-6}{7-1}=\frac{3}{6}=\frac{1}{2}$ $m_{\overline{AD}}=\frac{6-1}{1-3}=-\frac{5}{2}$

$$m_{\overline{CD}}=\frac{6-1}{13-3}=\frac{5}{10}=\frac{1}{2}\quad m_{\overline{BC}}=\frac{9-6}{7-13}=-\frac{3}{6}=-\frac{1}{2}$$

REF: 080134b

8 ANS:



. To prove that *KATE* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{AK}}=\frac{7-5}{4-1}=\frac{2}{3}$ $m_{\overline{EK}}=\frac{-1-5}{1-1}=$ **undefined**

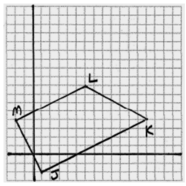
$$m_{\overline{ET}}=\frac{3-(-1)}{7-1}=\frac{4}{6}=\frac{2}{3}\quad m_{\overline{AT}}=\frac{7-3}{4-7}=-\frac{4}{3}$$

To prove that a trapezoid is not an isosceles trapezoid, show that the opposite sides that are not parallel are also not congruent using the distance formula: $\mathbf{d}_{\overline{EK}} = \sqrt{(1-1)^2 + (5-(-1))^2} = 6$

$$\mathbf{d}_{\overline{AT}} = \sqrt{(4-7)^2 + (7-3)^2} = 5$$

REF: 010333b

9 ANS:



. To prove that *JKLM* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{JK}} = \frac{4-(-2)}{13-1} = \frac{1}{2}$ $m_{\overline{JM}} = \frac{-2-4}{1-(-2)} = -2$

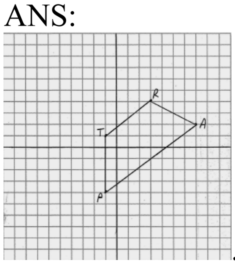
$$m_{\overline{LM}} = \frac{8-4}{6-(-2)} = \frac{1}{2} \quad m_{\overline{KL}} = \frac{4-8}{13-6} = -\frac{4}{7}$$

To prove that a trapezoid is not an isosceles trapezoid, show that the opposite sides that are not parallel are also not congruent using the distance formula: $d_{\overline{JM}} = \sqrt{(1-(-2))^2 + (-2-4)^2} = \sqrt{45}$

$$d_{\overline{KL}} = \sqrt{(13-6)^2 + (4-8)^2} = \sqrt{65}$$

REF: 080434b

10 ANS:



. To prove that *TRAP* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{\text{TR}}} = \frac{1-4}{-1-3} = \frac{3}{4}$ $m_{\overline{\text{TP}}} = \frac{1-(-4)}{-1-(-1)} = \textbf{undefined}$

$$m_{\overline{PA}} = \frac{-4-2}{-1-7} = \frac{3}{4} \quad m_{\overline{RA}} = \frac{4-2}{3-7} = -\frac{1}{2}$$

To prove that a trapezoid is not an isosceles trapezoid, show that the opposite sides that are not parallel are also not congruent using the distance formula: $d_{\overline{\text{TP}}} = \sqrt{(-1-(-1))^2 + (1-(-4))^2} = 5$

$$d_{\overline{RA}} = \sqrt{(3-7)^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5}$$

REF: 080933b

11 ANS:

To prove that *ABCD* is a trapezoid, show that one pair of opposite sides of the figure is parallel by showing they have the same slope and that the other pair of opposite sides is not parallel by showing they do not have the same slope: $m_{\overline{AB}} = \frac{0-0}{-a-a} = \frac{0}{-2a} = 0$ $m_{\overline{AD}} = \frac{c-0}{-b-(-a)} = \frac{c}{-b+a}$ If $\overline{AD}$ and $\overline{BC}$ are parallel,

$$m_{\overline{CD}} = \frac{c-c}{-b-b} = \frac{0}{-2b} = 0 \quad m_{\overline{BC}} = \frac{c-0}{b-a} = \frac{c}{b-a}$$

then: $\frac{c}{-b+a} = \frac{c}{b-a}$ But the facts of the problem indicate $a \neq b$, so $\overline{AD}$ and $\overline{BC}$ are not parallel.

$$c(b-a) = c(-b+a)$$
$$b-a = -b+a$$
$$2a = 2b$$
$$a = b$$

To prove that a trapezoid is an isosceles trapezoid, show that the opposite sides that are not parallel are congruent using the distance formula: $d_{\overline{BC}} = \sqrt{(b-a)^2+(c-0)^2}$ $d_{\overline{AD}} = \sqrt{(-b-(-a))^2+(c-0)^2}$

$$d_{\overline{BC}} = \sqrt{b^2-2ab+a^2+c^2} = \sqrt{a^2+b^2-2ab+c^2}$$

$$d_{\overline{AD}} = \sqrt{(a-b)^2+c^2} = \sqrt{a^2-2ab+b^2+c^2} = \sqrt{a^2+b^2-2ab+c^2}$$

REF: 080534b